SYNCHRONIZATION FOR TWO CHAOTIC FINANCE SYSTEMS

SINCRONIZAREA A DOUĂ SISTEME HAOTICE DIN FINANȚE

***OANCEA Servilia*[1], *BODALE I.*[2]**
e-mail: liaoancea@yahoo.com

***Abstract**. Financial systems can exhibit chaotic behaviour and this fact is very important for economic structures. In order to formulate the chaos control in finance, analyzing the dynamics of a chaotic financial system, the synchronization of two systems based on the adaptive feedback method of control is presented in this work. The transient time until synchronization depends on initial conditions of two systems, the strength and number of the controllers.*
***Key words**: finance system, chaos control*

***Rezumat**. Sistemele economice pot avea comportare haotica si acest fapt este foarte important pentru structurile economice. Pentru a realiza controlul haosului în sistemele financiare, în această lucrare sincronizăm două sisteme haotice, folosind o metodă de feedback. Timpul după care se obține sincronizarea depinde de condițiile initiale ale celor doua sisteme, de intensitatea si numarul functiilor de control.*
***Cuvinte cheie:** sistem financiar, controlul haosului*

INTRODUCTION

Financial systems can exhibit chaotic behaviour and this fact is very important for economic structures. From this point of view the deliberate control of these phenomena have a great practical impact despite the fact that it is very dificult; this is the reason the theoretical models are useful in these situations. Over the last decade, there has been considerable progress in generalizing the concept of synchronization to include the case of coupled chaotic oscillators especially from economical reasons. When the complete synchronization is achieved, the states of both systems become practically identical, while their dynamics in time remains chaotic. Many examples of synchronization have been documented in the literature, but currently theoretical understanding of the phenomena lags behind experimental studies (Grosu I., 1997), (Grosu I., et al, 2008), (Hu M., et al, 2008), (Lerescu A.I., et al, 2004), (Lerescu A.I. et al, 2006), (Oancea S., 2009). The main aim of this paper is to study the synchronization of two chaotic systems based on the adaptive feedback method of control.

[1] University of Agricultural Sciences and Veterinary Medicine of Iaşi, Romania
[2] "*Alexandru Ioan Cuza*" University of Iaşi, Romania

THEORY

One of the chaotic finance three-dimensional model is given by a nonlinear system of equations (Zhao X.et al., 2011)

$$\dot{x}_1 = x_3 + x_1(x_2 - a) \qquad (1)$$
$$\dot{x}_2 = 1 - bx_2 - x_1^2$$
$$\dot{x}_3 = x_1 - cx_3$$

In this model x denotes the interest rate, y denotes the investment demand and z denotes the price index. The parameter a denotes the savings, b denotes the investment cost and c denotes the commodities demand elasticity.

This system has a chaotic behaviour, for the following constants:

a=0.9, b=0,2, c=1,2 (2)

Figure 1 shows that the attractor projected onto $x_1x_2\ x_3$ space for the chaotic system (1) with values from (2).

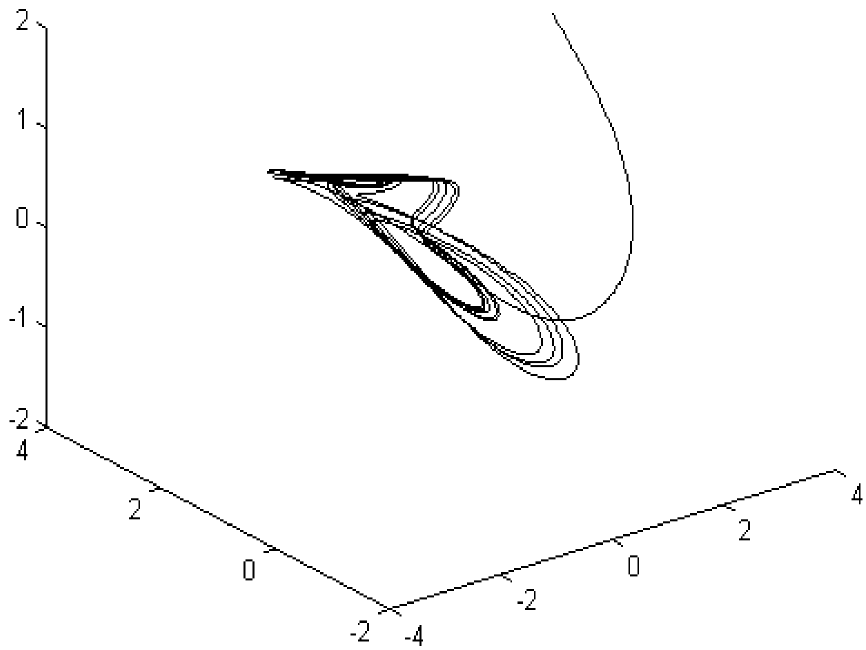

Fig. 1– Phase portrait of (x_1, x_2, x_3) for system (1)

To synchronize two chaotic systems we used a simple method for chaos synchronization proposed by Guo and coworkers (Guo W., et al, 2009) and used by Oancea (Oancea S., 2009).

If the chaotic system (master) is:

$\dot{x} = f(x)$ where

$x = (x_1, x_2, ...x_n) \in R_n$;

$f(x) = (f_1(x), f_2(x).....f_n(x)) : R^n \rightarrow R^n$

then the slave system is:

$\dot{y} = f(y) + z(y - x)$

where the functions

$\dot{z}_i = -\lambda_i (y_i - x_i)^2$

and λ_i are positive constants

RESULTS AND DISCUSSION

According this method of synchronization, the slave system for the system (1) will be:

$$\begin{aligned} \dot{y}_1 &= y_3 + y_1(y_2 - a) + z_1(y_1 - x_1) \\ \dot{y}_2 &= 1 - by_2 - y_1^2 + z_2(y_2 - x_2) \\ \dot{y}_3 &= y_1 - cy_3 + z_3(y_3 - x_3) \end{aligned} \tag{3}$$

and for the control strength:

$$\begin{aligned} \dot{z}_1 &= -(y_1 - x_1)^2 \\ \dot{z}_2 &= -(y_2 - x_2)^2 \\ \dot{z}_3 &= -(y_3 - x_3)^2 \end{aligned} \tag{4}$$

Figure 2 and 3 demonstrate the syncronization of the two finance systems.

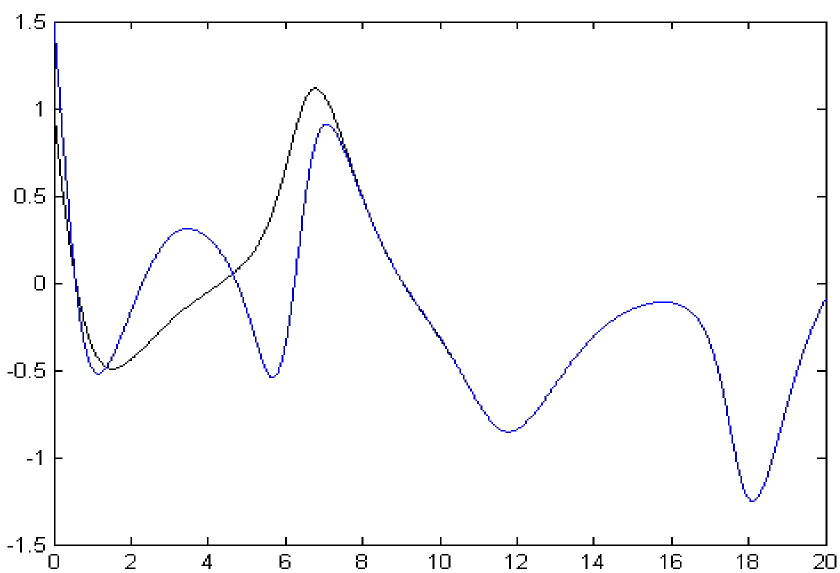

Fig. 2 – The synchronization of the two chaotic systems

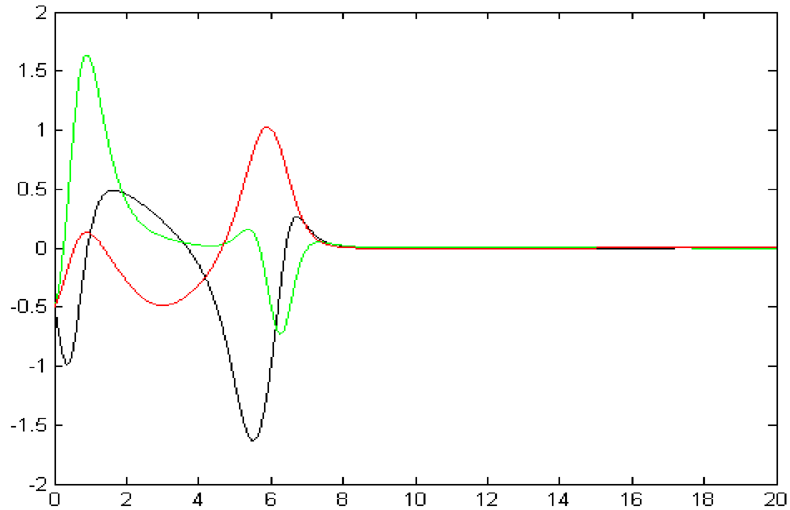

Fig. 3 – Synchronization errors between master and slave

Debin Huang (Huang D., 2005), by testing the chaotic systems including the Lorenz system, Rossler system, Chua's circuit, and the Sprott's collection of the simplest chaotic flows found that we can use a single controller to achieve identical synchronization of a three-dimensional system (for Lorenz system this is possible only we add the controller in the second equation). For the two finance systems we achieved the synchronization if one controller is applied only in the first or in the second equation (fig. 3,4 and 5).

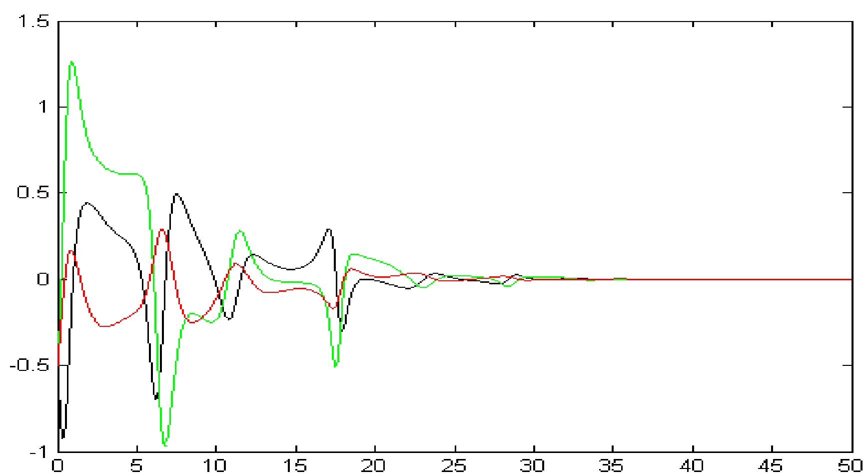

Fig. 4 – Synchronization errors between master and slave when the controller is applied in the first equation

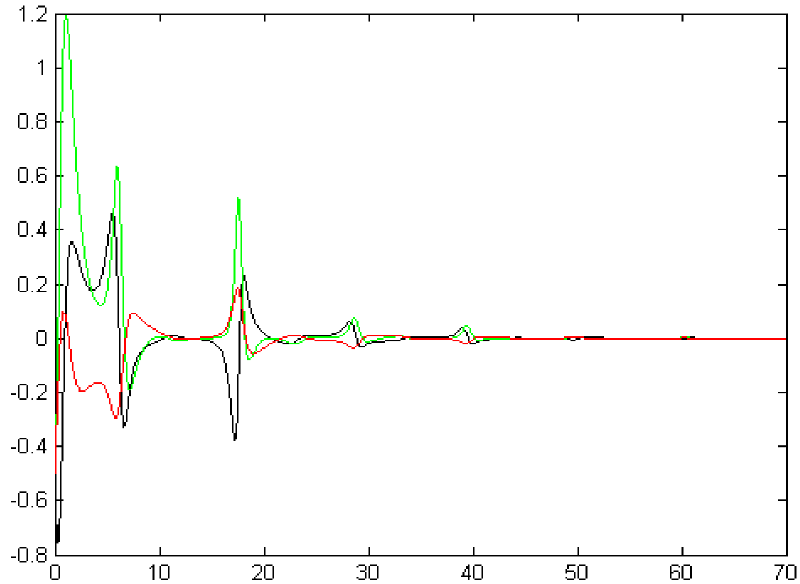

Fig. 5 – Synchronization errors between master and slave when the controller is applied in the second equation

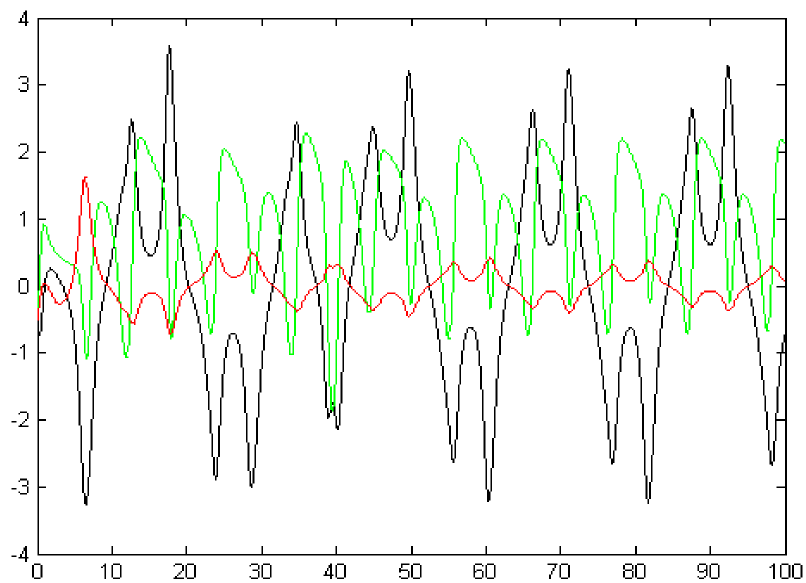

Fig. 6 – Synchronization errors between master and slave when the controller is applied in the third equation

CONCLUSIONS

In this paper we investigated the synchronization of two chaotic finance three-dimensional systems using an adaptive feedback method. The transient time until synchronization depends on initial conditions of two systems, the strength of the controllers and their number. We achieved the synchronization if one controller is applied only in the first or in the second equation, that means in interest rate and investment demand domains. Then we can control this finance system in accordance with recent debates of Wang and Chen (Wang J-W and Chen A-M., 2010) about full global synchronization and partial synchronization in a system of two or three coupled chaotic oscillators. The control method described in this paper is very easy and might be useful in the case of the other chaotic systems.

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